

(REVISED COURSE)

(3 Hours)

[Total Marks : 100

N.B. (1) Question No. 1 is compulsory.

(2) Attempt any four questions from Question Nos. 2 to 6.

1. (a) State total probability theorem and Bay's Theorem. 4
 (b) State and prove any two properties of— 8
 (i) Density functions
 (ii) Distribution functions.
 (c) If $R(\tau)$ is an autocorrelation function then prove that $R(\tau)$ is an even function. 4
 (d) Define Markov Chain giving an example. 4
2. (a) A mechanism consists of three paths A, B, C and probabilities of their failure are p, q, r respectively. The mechanism works if there is no failure in any of these parts. Find the probability that— 8
 (i) The mechanism is working
 (ii) The mechanism is not working.
 (b) If X, Y are two independent exponentially distributed random variables with same parameter unity, find the probability density function of 12
 $U = X + Y$
 $V = X/(X + Y)$.
3. (a) A random variable takes values 9, 13, 17 $(5 + 4n)$ each with probability $1/n$, find mean and variance of X. 8
 (b) The joint probability density function of (X, Y) is given by 12
 $f_{XY}(x, y) = K e^{-(X+Y)} 0 < x < y < \infty$.
 Find : (i) K
 (ii) Marginal densities of X and Y
 (iii) Are X, Y independent ?
4. (a) State and prove the properties of autocorrelation function and cross correlation function. 10
 (b) The power spectral density of random process is given by :— 10

$$S(\omega) = \frac{10\omega^2 + 35}{(\omega^2 + 4)(\omega^2 + 9)}$$

Find :—

- (i) Average Power
 (ii) $R(\tau)$ the autocorrelation function.

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5. (a) If the WSS process $X(t)$ is given by 12

$$X(t) = 10 \cos(100t + \theta)$$
 Where θ is uniformly distributed over $(-\pi, \pi)$.
 Prove that the $X(t)$ is correlation Ergodic.
- (b) Explain Power Spectral Density function. 8
 State its important properties and prove any one property.
6. (a) State and prove Chapman—Kolmogorov Equation. 8
 (b) (i) Define Central Limit Theorem and give its significance 12
 (ii) Define Strong Law of large numbers.
 (iii) Describe sequence of random variables.
7. (a) A medical representative visits only three cities A, B, C but he never visits 12
 the same city on successive days. If he visits city A today, then he visits
 city B tomorrow without fail. However, if he visits either city B or C today,
 then he is twice as likely to visit city A as the other city.
 In what proportion does he visit the cities A, B, C in the steady state.
- (b) X is continuous random variables with probability density function. 8

$$f_x(x) = (1/b)e^{-(x-a)/b} \quad ; \quad x > a$$

$$= 0 \quad ; \quad x < a$$
 Find characteristic function $\Phi_x(\omega)$ and hence determine the expected value
 of X .