Con. 5950-09. **(REVISED COURSE)** **SP-7739**

(3 Hours) **[Total Marks : 100**

N.B. (1) Question No.**1** is **compulsory**.
(2) Attempt any **four** questions out of remaining **six** questions.
(3) Make **suitable** assumptions if **required** and **justify** the **same**.
(4) **Figures** to the **right** indicate **full** marks.

1. (a) Derive one dimensional wave equation stating all conditions. 5
(b) Obtain the half range Cosine series for f (x) = x in (0, 2). 5
(c) A discrete random variable X has p.d.f. defined by — 5

X	–2	–1	0	1	2	3
P(X = x)	0·2	k	0·1	2k	0·1	2k

Obtain k, mean and variance.

(d) The two regression lines are given by :— 5
$20x - 9y - 107 = 0$ and $4x - 5y + 33 = 0$,
Obtain — (i) Coefficient of correlation between x and y
(ii) Mean values of x and y.

2. (a) Obtain complex form of Fourier series for the function — 6
$f(x) = e^{-x}$ in (–1, 1)
(b) A box contains **a** white and **b** black balls. **C** balls are drawn at random from the box. Find the expected value of number of white balls drawn. 6
(c) The diameter of a semicircular plate of radius **a** is kept at 0°C and temperature at semicircular boundary is T°C. Find the steady state temperature in the plate. 8

3. (a) Fit a Poisson distribution to the following data and test for goodness of fit :— 8

X	0	1	2	3	4	5
Frequency	223	142	48	15	4	0

(b) Obtain Fourier series for — 6

$$f(x) = \frac{\pi}{2} + x, \quad -\pi < x < 0$$

$$= \frac{\pi}{2} - x, \quad 0 < x < \pi.$$

(c) Fit a second degree parabolic curve to the following data :— 6

X	1	2	3	4	5	6	7	8	9
Y	2	6	7	8	10	11	11	10	9

4. (a) Probability that an electronic component will fail in less than 1200 hrs of continuous use is 0·25. Using normal approximation to Binomial distribution find the probability that among 200 such components less than 45 will fail in less than 1200 hrs of continuous use. 6
(b) A tightly stretched string with fixed end points x = 0 and x = l is initially in a position given by $y(x, 0) = y_0 \sin^3\left(\frac{\pi x}{l}\right)$. If it is released from rest from this position find displacement y at any distance x from one end at any time t. 6

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(c) Obtain Fourier series for $f(x) = x + x^2$; $-\pi \le x \le \pi$ and $f(x + 2\pi) = f(x)$. **8**

Hence deduce that $\frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots\cdots$

5. (a) Using method of separation of variables solve $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ **6**

$U = 0$ as $t \to \infty$; $u = 0$ when $x = 0$ and $x = L$.

(b) A random sample of 50 items has mean 6·2 and variance 10·24. Can it be regarded as drawn from a normal population with mean 5·4 at 5% level of significance ? **6**

(c) What is rank correlation ? Determine the coefficient of rank correlation from the following data :— **8**

X	10	12	18	18	15	40
Y	12	18	25	25	50	25

6. (a) Find Fourier integral representation of $f(x) = 1 - x^2$; $-1 \le x \le 1$ and is zero otherwise. **6**

(b) Two independent samples of sizes 8 and 7 gave the following results :— **6**

Sample 1	19	17	15	21	16	18	16	14
Sample 2	15	14	15	19	15	18	16	

Is the difference between sample means significant ?

(c) Obtain two lines of regression and coefficient of correlation from the following data :— **8**

X	65	66	67	67	68	69	70	72
Y	67	68	65	66	72	72	69	71

7. (a) Prove that the set of functions $\cos nx$; $n = 1, 2, \ldots\ldots$ is orthogonal over $(-\pi, \pi)$. Hence construct corresponding orthonormal set. **6**

(b) A die was thrown 132 times and following frequencies were obtained — **6**

Number obtained	1	2	3	4	5	6
Frequency	15	20	25	15	29	28

Test whether the die is unbiased.

(c) Obtain all possible solutions of two dimensional heat equation for steady state conditions using method of separation of variables. **8**