Con. 5945-09. **(REVISED COURSE)** **SP-7757**

(3 Hours) [Total Marks : 100

N.B. : (1) Question No. **1** is **compulsory**.
(2) Solve any **four** from the **remaining**.
(3) **All** the questions carry **equal** marks.
(4) **Data table** will be given, if **required**.

1. (a) If ϕ and ψ are scalar point functions, then prove that $\nabla\phi \times \nabla\psi$ is solenoidal. **5**

(b) Determine, algebraic and geometric multiplicity of A, where $A = \begin{bmatrix} 1 & 2 & 2 \\ 0 & 2 & 1 \\ -1 & 2 & 2 \end{bmatrix}$ **5**

(c) Solve $3x + 3y + 2z = 1$, $x + 2y = 4$, $10y + 3z = -2$, $2x - 3y - z = 5$. **5**

(d) Define conditional probability. If $P(A) = P_1$ $P(B) = P_2$ show that $P(A/B) \geq \frac{P_1 + P_2 - 1}{P_2}$. **5**

2. (a) Using Green's Theorem find the Area of the region in the first Quadrant bounded by the curves $y = x$, $xy = 1$, $4y = x$. **6**

(b) Box I contains 10 white and 3 black balls. Box II contains 3 white and 5 black balls. Two balls are drawn at random and placed in box II. Then one ball is drawn at random from box II. What is the probability that it is a white ball ? **6**

(c) If $A = \begin{bmatrix} 1 & 3 & -1 & 2 \\ 0 & 11 & -5 & 3 \\ 2 & -5 & 3 & 1 \\ 4 & 1 & 1 & 5 \end{bmatrix}$. **8**

Reduce A to Echelon form and then row Canonical form. Hence find the rank of A.

3. (a) Find first four Central Moments for Poisson Distribution using Recurrence relation. **6**

(b) For $\overline{F} = (x - y)\, i + (x + y)\, j$, Evaluate $\int \overline{F} \cdot d\overline{r}$ around the curve C consisting of $y = x^2$ and $y^2 = x$. **6**

(c) Find the matrix P which diagonalises the matrix. $A = \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}$. Verify that $P^{-1}AP = D$, where D is the diagonal matrix. Hence find A^6. **8**

4. (a) Evalaute $\int_c f\, d\overline{r}$, where $f = 2xy^2z + x^2y$ and C is the curve $x = t$, $y = t^2$, $z = t^3$ from $t = 0$ to $t = 1$. **6**

(b) Find the mean and standard deviation of a Normal Distribution in which 7% of the terms are under 35 and 89% are under 63. **6**

(c) Show that $\begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$ is diagonisable. Find the transformation matrix and the diagonal matrix. **8**

5. (a) If O is the acute angle between the lines of regression, prove that— **6**

$$\tan\theta = \left(\frac{1-r^2}{r}\right)\left(\frac{\sigma_x - \sigma_y}{\sigma_x^2 + \sigma_y^2}\right),$$ where r, σ_x, σ_y have their usual meaning.

(b) A box contains 'a' white balls and 'b' black balls. Now C balls are drawn at random from the box. Find the expected value of the number of white balls. **6**

(c) Verify Guass Divergence Theorem for $\overline{F} = 2x^2y\, i - y^2\, j + 4x\, z^2k$ taken over the region in the first octant bounded by the cylinder $y^2 + z^2 = 9$ and the plane $x = 2$. **8**

6. (a) Reduce the Quadratic form $3x_1^2 + 2x_2^2 + x_3^2 + 4x_1x_2 + 6x_2x_3 - 2x_1x_3$ to sum of squares and find the corresponding linear transformation. **6**

(b) Find the value of 'b' such that rank of A is 3 where $A = \begin{bmatrix} 1 & 1 & -1 & 0 \\ 4 & 4 & -3 & 1 \\ b & 2 & 2 & 2 \\ 9 & 9 & b & 3 \end{bmatrix}$. **6**

(c) (i) Show that $\overline{F} = \frac{\overline{r}}{r^3}$ is Irrotational and Solenoidal. **4**

(ii) Show that $\nabla^2 f(r) = f''(r) + \frac{2}{r} f'(r)$. **4**

7. (a) For $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$

(i) Verify Caley Hamilton theorem. **3**

(ii) Find the matrix equation. **3**

$$A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I.$$

(b) A continuous random variable x has p.d.f. $f(x) = kx^2, 0 \le x \le 2$. Determine k, $P(0{\cdot}2 \le x \le 0{\cdot}5)$ and $P(x \ge 0{\cdot}75/x \ge 0{\cdot}5)$. **6**

(c) If x and y are uncorrelated random variables, then show that coefficient correlation between x + y and x – y, that is $r(x+y, x-y) = \frac{\sigma_x^2 - \sigma_y^2}{\sigma_x^2 + \sigma_y^2}$. **8**