

(3 Hours)

[Total Marks : 100

- N.B. (1) Question No.1 is compulsory.
 (2) Attempt any four questions out of the remaining six questions.
 (3) Figures to the right indicate full marks.

1. (a) Prove that the eigen values of an orthogonal matrix are +1 or -1. 5
 (b) Show that a harmonic function 'u' satisfies the differential equation $\frac{\partial^2 u}{\partial z \partial \bar{z}} = 0$. 5
 (c) $\vec{F} = (x + 2y + az)\vec{i} + (bx - 3y - z)\vec{j} + (4x + cy + 2z)\vec{k}$ if \vec{F} is irrotational then find 5
 (i) the constants a, b, c (ii) a scalar function ϕ such that $\vec{F} = \nabla Q$.
 (d) Prove that $J_n(x)$ is an even function if n is an even number and is an odd function 5
 if n is an odd number.
2. (a) If $f(z) = z^n$ then show that $f(z)$ is an analytic function and hence find $f'(z)$. 6
 (b) Show that the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ satisfies Cayley-Hamilton Theorem and hence 7
 find A^{-1} if exists.
 (c) Expand $f(x) = 1$ in $0 < x < 1$ in a series as $1 = \sum_{i=1}^{\infty} \frac{2}{\lambda_i J_1(\lambda_i)} J_0(\lambda_i x)$ 7
3. (a) Find the analytic function whose real part is $\frac{\sin(2x)}{\cosh(2y) + \cos(2x)}$ 6
 (b) Find eigen values and eigen vectors of A^3 where $A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$. Is A derogatory? 7
 (c) Evaluate $\int_C \vec{F} \times d\vec{r}$ where $\vec{F} = (2xy + z^2)\vec{i} + x^2\vec{j} + 3xz^2\vec{k}$ along the curve $x = t, y = t^2,$ 7
 $z = t^3$ from $(0, 0, 0)$ to $(1, 1, 1)$.
4. (a) Expand $f(z) = \frac{1}{z(z+1)(z-2)}$ in Laurent's series when — 6
 (i) $|z| < 1$ (ii) $1 < |z| < 2$ (iii) $|z| > 2$
 (b) Evaluate $\int_0^{\pi} \frac{d\theta}{3 + 2 \cos \theta}$ 7

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- (c) Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ 1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ 7
5. (a) Evaluate $\int_C \frac{z^2}{z^4 - 1} dz$ where C is the circle — 6
- (i) $|z| = \frac{3}{4}$ (ii) $|z - 1| = 1$ (iii) $|z + i| = \frac{1}{2}$
- (b) Evaluate $\int_0^{Hi} (x^2 + iy) dz$ along the path (i) $y = x$ (ii) $y = x^2$. Is the line integral 7
independent of the path.
- (c) Verify Stoke's Theorem for $\vec{F} = y\vec{i} + z\vec{j} + x\vec{k}$ over the surface $x^2 + y^2 = 1 - z$, ($z > 0$) 7
6. (a) Prove that $J_{-\frac{3}{2}}(x) = -\sqrt{\frac{2}{\pi x}} \left[\frac{\cos(x)}{x} + \sin(x) \right]$ 6
- (b) Find the bilinear transformation which maps the points 2, i, -2 on to the points 1, i, -1. 7
Is this transformation parabolic?
- (c) Reduce the following quadratic form $Q = 2x^2 + y^2 - 3z^2 - 8yz - 4xz$ to normal form 7
through congruent transformations. Also find its rank and signature.
7. (a) Find 4^A if $A = \begin{bmatrix} \frac{3}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{3}{2} \end{bmatrix}$ 6
- (b) Find the residues at singular points of $f(z) = \frac{z}{(z-1)^2(z^2-1)}$ and hence evaluate 7
 $\int_C f(z) dz$ where C is $|z| = 2$.
- (c) Using Green's Theorem evaluate $\int_C (e^{x^2} - xy) dx - (y^2 - x) dy$ where C is the 7
circle $x^2 + y^2 = 1$.