II stpalf-09-nkF-G 30 Sem III / Rev / Prod'n / App. maths III / Jun-10

Con. 6055-09. **SP-7332**

(REVISED COURSE)

(3 Hours) **[Total Marks : 100**

N.B.:(1) Question No. **1** is **compulsory**.
(2) Attempt any **four** questions out of the remaining **six** questions.

1. (a) Evaluate $\int_0^\infty \frac{e^{-t}[\cos(3t) - \cos(2t)]}{t}\, dt$ **5**

(b) Find the matrix A if **5**

$$\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} A \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} -2 & 4 \\ 3 & -1 \end{bmatrix}.$$

(c) Find a Fourier series to represent $f(x) = x^2$ in $(0, 2\pi)$ **5**

(d) Find the image of $|z - a_i| = a$ under the transformation $w = 1/2$. **5**

2. (a) Test for consistency and if possible solve **6**

$x + y + z = 6, \quad x - y + 2z = 5, \quad 3x + y + z = 8, \quad 2x - 2y + 3z = 7$

(b) Prove that $\int_0^\infty \left[\frac{\sin(2t) + \sin(3t)}{t\, e^t}\right] dt = \frac{3\pi}{4}$. **6**

(c) Obtain Fourier series for the function— **8**

$$f(x) = \begin{cases} 1 + \dfrac{2x}{\pi} & -\pi \le x \le 0 \\ 1 - \dfrac{2x}{\pi} & 0 \le x \le \pi \end{cases}$$

Deduce that—

$$\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots\dots$$

3. (a) Find the eigen values and eigen-vectors of the matrix **6**

$$\begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$$

(b) Find the bilinear transformation under which 1, i, – 1 from the z-plane are mapped onto 0, 1, ∞ of w-plane. **6**

(c) Solve the following equation by using Laplace transforms **8**

$$\frac{dy}{dt} + 2y + \int_0^t y\,dt = \sin t \quad \text{given that } y(0) = 1$$

[TURN OVER

4. (a) Find $L^{-1}\left[\operatorname{Tan}^{-1}\left(\frac{2}{s^2}\right)\right]$ **6**

(b) Find non-singular matrices P and Q such that— **6**

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \\ 3 & 0 & 5 & -10 \end{bmatrix}$$

(c) Find the Fourier series of $f(x) = \begin{cases} x & 0 \le x \le \pi \\ 2\pi - x & \pi < x \le 2\pi \end{cases}$ **8**

Hence deduce that $\frac{\pi^4}{96} = \frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \ldots\ldots\ldots$

5. (a) Show that every square matrix can be uniquely expressed as the sum of Hermitian and Skew Hermitian matrix. **6**

(b) Find an analytic function $f(z) = u + iv$ where $u - v = e^x(\cos y - \sin y)$ find $f(z)$ in terms of z. **6**

(c) Find the orthogonal trajectory of the family of curves $e^x \cos y - xy = c$ and hence show that the obtained curve is harmonic. **8**

6. (a) Find $L^{-1}\left[\frac{(s+2)^2}{(s^2+4s+8)^2}\right]$ using convolution theorem. **6**

(b) Evaluate $\int_C \log z\,dz$ where C is a unit circle in the z-plane. **6**

(c) Show that $w = i\left(\frac{1-z}{1+z}\right)$ transforms the circle $|z| = 1$ onto the real axis of the w-plane and the interior of the circle $|z| < 1$ onto the upper half of the w-plane. **8**

7. (a) Find L·T of $t\,e^{3t}\operatorname{erf}(\sqrt{t})$ **6**

(b) Show that $L\left[\frac{\cos(\sqrt{t})}{\sqrt{t}}\right] = \sqrt{\pi/s}\; e^{-1/4s}$ **6**

(c) Obtain half range sine series for f(x) when **8**

$$f(x) = \begin{cases} x & 0 < x < \pi/2 \\ \pi - x & \pi/2 < x < \pi \end{cases}$$

Hence deduce that—

$$\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots\ldots\ldots$$
