Sem III / Rev / mech / APP. maths III / Jan -10

Con. 6126–09. **(REVISED COURSE)** **SP–7298**

(3 Hours) **[Total Marks : 100**

N.B. 1. Question No. 1 is compulsory
2. Attempt any FOUR from remaining six questions.
3. Figures to the right indicate the full marks.
4. Assume the suitable data if needed with justification.

Q.1 a. Find Laplace transform of 05

$$f(t) = \frac{\sin\sqrt{t}}{\sqrt{t}}$$

b. If λ is an Eigen value of 'A' then λ^n is an 05

eigen value of $\mathbf{A^n}$ corresponding to same Eigen vectors 'X'

where 'n' is a natural number.

c. Find the value of k if $\mathbf{f(z) = r^3 \cos k\theta + i r^k \sin k\theta}$ is analytic. 05

d. Prove that every hermitian matrix A can be expressed as 05
B + i C where B is real symmetric and C is real skew-symmetric matrix

Q.2 a. Find two non-singular matrices P & Q such that PAQ is in 06
normal form for,

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 3 & 1 & 1 \end{bmatrix}$$

b. Find the analytic function f (z) =u + i v if, 06

$$\mathbf{u - v = (x - y)(x^2 + 4xy + y^2)}$$

c. Find the Laplace transformation of, 08

(i) $\frac{1}{t}(\cos at - \cos bt)$ (ii) $e^{-3u}\int_0^t u \sin 3u\,du$

Q.3 a. Show that $\int_C \frac{dz}{z+1} = 2\pi i$ where C is the circle | z | = 2 06

Hence deduce that , $\int_C \frac{(x+1)\,dx + y\,dy}{(x+1)^2 + y^2} = 0$ and $\int_C \frac{(x+1)\,dy - y\,dx}{(x+1)^2 + y^2} = 2\pi$

b. Find the eigen value and eigen vectors of, 06

$$A = \begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$$

c.Find, 08

(i) $L^{-1}\left\{\frac{s^2 + 2s + 3}{(s^2 + 2s + 2)(s^2 + 2s + 5)}\right\}$ (ii) $L^{-1}\left\{s \log\left(\frac{s+1}{s-1}\right)\right\}$

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Q.4 a. Prove : 06

$$\int_0^{\infty} t e^{-3t} J_0(4t)\,dt = \frac{3}{125} \text{ where } L\{J_0(t)\} = \frac{1}{\sqrt{1+s^2}}$$

b. Define orthogonal matrix. 06

Prove that the following matrix is an orthogonal and hence find its inverse.

$$\begin{bmatrix} \cos\alpha & 0 & \sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & 0 & \cos\alpha \end{bmatrix}$$

c. Expand $f(z) = \dfrac{1}{(z+1)(z+3)}$ in Laurent's series valid for, 08

(i) $1 < |z| < 3$ about $z = 0$ (ii) $|z| > 3$ about $z = 0$

(iii) $0 < |z+1| < 2$ about $z = -1$ (iv) $|z| < 1$ about $z = 0$

Q.5 a. Using Convolution theorem find, 06

$$L^{-1}\left\{\frac{s^2}{(s^2+a^2)(s^2+b^2)}\right\}$$ Also, verify it.

b. $\oint_C \dfrac{z^2+z+1}{z^2-7z+2}\,dz$ where C is the curve $4x^2+9y^2+1$ 06

c. **Find the values of 'k' for which the equations** 08

$x+y+z=1$; $x+2y+3z=k$; $x+5y+9z=k^2$ have a solution. Also find it for these values of k

Q.6 a. For $A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}$ Find sin A 06

b. Solve : $3\dfrac{dy}{dt} + 2y = e^{3t}$ *where* $y = 1$ *at* $t = 0$ 06

c. Define (i) Billinear Transformation 08

(ii) Fixed point of a Billinear Transformation

and Find the bilinear transformation which maps the points 2, i, -2 onto the points 1, i, -1. Also find its fixed points.

Q.7 a. Define a periodic function and hence find its Laplace transformation 06

b. Examine whether the following vectors are linearly dependent or independent if they are dependent find the relation between them. 06

[1,0,2,1], [3,1,2,1], [4,6,2,–4], [–6,0,–3,–4]

c. If $A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$ verify Cayley-Hemilton thm. Find A^{-1} 08

and express $A^6 - 4A^5 + 8A^4 - 12A^3 + 14A^2$ as a linear polynomial in A.

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