Sem III / Rev / I.T. / App. Maths-III / Jan 10

Con. 5533-09. **(REVISED COURSE)** **SP-7433**

(3 Hours) **[Total Marks : 100**

N.B. : (1) Question No. 1 is **compulsory**.
(2) Attempt any **four** out of remaining **six** questions.
(3) Make **suitable** assumptions if **required** and **justify** the same.
(4) **Figures** to the **right** indicate **full** marks.

1. (a) Show that every square matrix can be uniquely expressed as the sum of a Hermitian matrix and a Skew-Hermitian matrix. **5**

(b) Show that the set of functions sin x, sin 2x, sin 3x, ... is orthogonal on $(0, 2\pi)$. **5**

(c) Find the Laplace transforms of the following :– **5**

(i) $t\sqrt{1+\sin t}$ (ii) $te^{3t}\sin t$

(d) Construct the analytic function whose real part is $e^{2x}(x\cos 2y - y\sin 2y)$. **5**

2. (a) Prove that $\int_0^\infty e^{-t}\frac{\sin^2 t}{t}\,dt = \frac{1}{4}\log 5$. **6**

(b) Find the Fourier expansion of $f(x) = \frac{1}{4}(\pi - x)^2$ in$(0,2\pi)$.. **8**

(c) Show that $u = y^3 - 3x^2y$ is a harmonic function. Find its harmonic conjugate and the corresponding analytic function. **6**

3. (a) Find the analytic function $f(z) = u + iv$ in terms of z if $u + v = \frac{x}{x^2+y^2}$. **6**

(b) Obtain Fourier series for **8**

$$f(x) = x + \frac{\pi}{2}, \quad -\pi < x < 0$$

$$= \frac{\pi}{2} - x, \quad 0 < x < \pi$$

Hence, deduce that $\frac{\pi^2}{96} = \frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \ldots\ldots\ldots\ldots$

(c) State Cauchy Reimann equation in Cartesian and Polar form. Determine the constant a, b, c, d if $f(z) = x^2 + 2axy + by^2 + i(cx^2 + 2dxy + y^2)$ is analytic. **6**

4. (a) Find the Laplace transform of the following :– **6**

(i) $\int_0^t u\cos^2 u\,du$ (ii) $\int_0^t \frac{1-e^{-an}}{u}\,du$

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(b) Reduce A to normal form and find its rank where 6

$$A = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 4 & 6 & 8 & 10 \\ 15 & 27 & 39 & 51 \\ 6 & 12 & 18 & 24 \end{bmatrix}$$

(c) Obtain half range cosine series for $f(x) = x - x^2$, $0 \leq x \leq 1$. 8

5. (a) Find inverse Laplace transform of the following :– 8

(i) $\dfrac{1}{(s-2)(s+2)^2}$ (ii) $\log \dfrac{s+a}{s+b}$

(b) Use the adjoint method to find the inverse of A where 6

$$\begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & 2 \end{bmatrix}$$

(c) If $f(a) = \int_C \dfrac{4z^2 + z + 5}{z - a}\, dz$, where C is $|z| = 2$, find the values of 6

$f(1)$, $f(i)$, $f(-1)$, $f(-i)$.

6. (a) Solve using Laplace transform $\dfrac{d^2y}{dt^2} + 9y = 18t$ given that $y(0) = 0$ & $y\left(\pi/2\right) = 0$. 8

(b) Solve the following set of homogeneous equations 6

$x + 2y + 3z = 0$, $2x + 3y + z = 0$, $4x + 5y + 4z = 0$, $x + 2y - 2z = 0$

(c) Evaluate $\int_C \dfrac{z-1}{(z+1)^2 (z-2)}\, dz$, where C is $|z-i| = 2$. 6

7. (a) Expand $f(z) = \dfrac{1}{z(z+1)(z-2)}$ 8

(i) within the unit circle about the origin.

(ii) within the annulus region between the concentric circles about the origin having radii 1 and 2 respectively.

(iii) in the exterior of the circle with center at the origin and radius 2.

(b) Solve the equations 6

$x + y + z = 3$, $x + 2y + 3z = 4$, $x + 4y + 9z = 6$.

(c) Find the inverse Laplace transform of the following :– 6

(i) $\dfrac{e^{4-3s}}{(s+4)^{5/2}}$ (ii) $\dfrac{8e^{-3s}}{s^2 + 4}$