Con. 5932-09. (REVISED COURSE) SP-7322

(3 Hours) [Total Marks :100

N.B. (1) Question No. **1** is **compulsory.**
(2) Attempt any **four** questions from the **remaining** questions.
(3) **All** the questions carry **equal** marks.

1. (a) Find the Laplace transform of $\int_t^{\infty} \frac{\text{Cos}\,u}{u}\,du$. 20

(b) Find the analytic function f(z) = u + iv where $v = \tan^{-1}\frac{y}{x}$

(c) Find the image of $x^2 - y^2 = 1$ under $w = \frac{1}{z}$. Interpret with sketches.

(d) Obtain Fourier series of $f(x) = x - x^2$ in $-1 < x < 1$.

2. (a) Find the Fourier series of $f(x) = 1 + \frac{2x}{\pi}, \ -\pi \le x \le 0$ 6

$= 1 - \frac{2x}{\pi}, \ 0 \le x < \pi$

and deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots\cdots = \frac{\pi^2}{8}$

(b) Show that w = Sin 2 is conformal every where except at $z = \pm\frac{\pi}{2}$ 6

(c) Find :—

(i) $L\left[\frac{1-\text{Cost}}{t^2}\right]$ 4

(ii) $L\left[t^{-1}\int_0^t e^{-u}\ \text{Sinu}\ du\right]$ 4

3. (a) Find Half Range Sine Series for $f(x) = lx - x^2$, $0 < x < l$ and deduce that – 6

$\frac{1}{1^3} - \frac{1}{3^3} + \frac{1}{5^3} - \frac{1}{7^3} + \cdots = \frac{\pi^3}{3^2}$

(b) Using Laplace transform, solve :— 6
$y'' + n^2y = a\ \text{Sin}\ (nt + 2)$, where (0) = 0 = y′ (0)

(c) Expand $f(z) = \frac{1}{z^2 - 3z + 2}$ in the regions 8

(i) $|z| < 1$ (ii) $1 < |z| < 2$ (iii) $|z| > 2$ (iv) $0 < |z-1| < 1$

4. (a) Sketch the curve for f(x) = 0 $\quad -2 < x < -1$ 6

$= 1 + x \quad -1 < x < 0$

$= 1 - x \quad 0 < x < 1$

$= 0 \quad 1 < x < 2$

and obtain Fourier series.

(b) State and prove necessary condition for any function f(z) to be analytic is a given region R. 6

(c) Evaluate :— 4

(i) $\int_0^{\pi}\frac{d\theta}{(2+\text{Cos}\,\theta)^2}$

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(ii) $\int_{-\infty}^{\infty} \frac{dx}{x^4+1}$ 4

5. (a) Find the bilinear transformation that maps z = 0, 1, ∞ onto w = i, –1, –i. 6

(b) Using Convolution theorem find the inverse Laplace transform of $\frac{1}{s(s+1)(s+2)}$ 6

(c) Find :—

(i) $L^{-1}\left[\frac{s}{s^4+s^2+1}\right]$ 4

(ii) $L^{-1}\left[S\log\frac{s}{\sqrt{s^2+1}} + \cot^{-1}s\right]$ 4

6. (a) Express $f(x) = \sin x,\ 0 < x \le \pi$ 6
$= 0$ else where,

as a Fourier integral and prove that $f(n) = \frac{1}{\pi}\int_0^{\infty} \frac{\sin wx + \cos[w(\pi - x)]}{1-w^2}\, dw$

Also deduce that $\int_0^{\infty} \frac{\cos\left(w\pi/2\right)}{1-w^2}\, dw = \frac{\pi}{2}$

(b) Find finite Fourier Sine and Cosine tranform of f(x) = π x - x², 0 < x < π . 6

(c) Evaluate :—

(i) $\int_c \frac{\sin z}{z^6}\, dz,\ c: |z| = 2$ 4

(ii) $\int_c \frac{dz}{\sin h2z},\ c: |z| = 2$ 4

7. (a) Find Fourier series of $f(x) = x\cos\left(\frac{\pi x}{L}\right), -L \le x \le L.$ 6

(b) Evaluate $\int_C f(z)dz$ where $f(z) = \begin{cases} 4y\ ; & y > 0 \\ 1\ ; & y < 0 \end{cases}$ and 6
c is the arc from z = –1 – i to z = 1 + i of the cubical curve y = x³.

(c) Find :—

(i) $L^{-1}\left[\frac{s^2}{(s^2+a^2)^2}\right]$ 4

(ii) Express f(t) in terms of Heaniside unit step function and find its Laplace transform. Where :— 4

f(t) =	Sint,	0 < t < π
=	Sin 2t	π < t < 2 π
=	Sin 3t	t > 2 π.