

- N. B. :** (1) Question No. 1 is **compulsory**.
 (2) Attempt any **four** questions out of remaining **six** questions.
 (3) Use of **calculator's** (Non-Programmable) is **allowed**.
 (4) Write the sub-questions of the **main** question collectively.
 (5) **Figures** to the **right** indicates **full** marks.

1. (a) Find the Z-transform of $f(k) = a^k, k \geq 0$. 5
 (b) Find $L(\operatorname{erf} \sqrt{t})$ 5
 (c) Is the following Matrix orthogonal? If not, can it be converted into an orthogonal matrix? If yes how? 5

$$A = \begin{bmatrix} 2 & 2 & 1 \\ -2 & 1 & 2 \\ 1 & -2 & 2 \end{bmatrix}$$

- (d) Find the half range cosine series for $f(x) = x, 0 < x < 2$. 5

2. (a) Evaluate $\int_0^{\infty} \frac{t^2 \sin 3t}{e^{2t}} dt$. 6
 (b) Obtain the Fourier Series 6

$$f(x) = x + \frac{\pi}{2} \quad -\pi < x < 0$$

$$= \frac{\pi}{2} - x \quad 0 < x < \pi$$

- (c) If A is a square matrix of order n and $|A| \neq 0$, then prove that— 8

(i) $|A^{-1}| = \frac{1}{|A|}$

(ii) $\operatorname{adj}(A^{-1}) = (\operatorname{adj} A)^{-1}$

3. (a) Find the Laplace Transformation of $\frac{\cos \sqrt{t}}{\sqrt{t}}$. 6
 (b) Show that every square matrix can be uniquely expressed as the sum of a Hermitian matrix and Skew-Hermitian matrix. 6
 (c) Find Fourier Sine integral representation for $f(x) = \frac{e^{-ax}}{x}$ 8

4. (a) Solve $\frac{d^2y}{dt^2} + 4y = f(t)$ with conditions $y(0) = 0, y'(0) = 1$ 6
 and $f(t) = 1$ when $0 < t < 1$
 $f(t) = 0$ when $t > 1$
- (b) Examine whether the vectors $X_1 = [3, 1, 1], X_2 = [2, 0, -1], X_3 = [4, 2, 1]$ are 7
 Linearly independent.
- (c) Examine a Fourier series to represent $f(x) = x^2$ in $(0, 2\pi)$ and hence deduce 7
 that, $\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots$
5. (a) Find Laplace transform of Sinhat. Sinat. 6
- (b) Find the Z - transform of $\text{Cos}\left(\frac{k\pi}{3} + a\right), k \geq 0.$ 6
- (c) Find the Fourier expansion of 8
 $f(x) = 2$ when $-2 < x < 0$
 $f(x) = x$ when $0 < x < 2$
6. (a) Show that functions $f_1(x) = 1, f_2(x) = x$ are orthogonal on $(-1, 1).$ 6
 Determine the constants a and b such that the function $f_3(x) = -1 + ax + bx^2$
 Is orthogonal to both f_1 and $f_2.$
- (b) State Convolution theorem for Z - transform. 6
 Hence find Z - transform of $f(k)*g(k)$ where $f(k) = 4^k U(k)$
 $g(k) = 5^k U(k)$
- (c) Find Inverse L.T. of 8
- (i) $\frac{s^2}{(s^2 - a^2)^2}$
- (ii) $\frac{1}{s\sqrt{s+4}}$
7. (a) Test the consistency of the equation and solve them if they are consistent. 6
 $2x - y + z = 9, 3x - y + z = 6, 4x - y + 2z = 7, -x + y - z = 4$
- (b) Obtain the complex form of Fourier series for $f(x) = \cosh ax$ in $(-L, L).$ 6
- (c) Find the L. T. of 8
- (i) $\int_0^t u^{-1} e^{-u} \sin u \, du$
- (ii) $\cosh t \int_0^t e^u \cosh u \, du.$