Sem III / Prod / 18-06-10 / Applied maths-III

Con. 3977-10. **AN-2476**

(3 Hours) **[Total Marks : 100**

N.B. : (1) Question No. **1** is **compulsory**.
(2) Attempt any **four** questions out of the remaining **six** questions.

1. (a) Evaluate $\int_{0}^{\infty} \frac{\cos(at) - \cos(bt)}{t}\,dt$. **5**

(b) If $A = \begin{bmatrix} 3 & 2 & 2 \\ 1 & 3 & 1 \\ 5 & 3 & 4 \end{bmatrix}$ find B such that **5**

$$AB = \begin{bmatrix} 3 & 4 & 2 \\ 1 & 6 & 1 \\ 5 & 6 & 4 \end{bmatrix}.$$

(c) Obtain Fourier series for $f(x) = x^2$ in $(-l, l)$ and hence deduce that **5**

$$\frac{\pi^2}{6} = \sum_{n=1}^{\infty} \frac{1}{n^2}$$

(d) Find the image of $|z - ai| = a$ under the transformation $w = \frac{1}{2}$. **5**

2. (a) Investigate for what values of λ and μ the equation $x + y + z = 6$, $x + 2y + 3z = 10$, $x + 2y + \lambda z = \mu$ have (i) no solution (ii) a unique solution (iii) infinite number of solutions. **6**

(b) Show that $L\left[\frac{\cos(\sqrt{t})}{\sqrt{t}}\right] = \sqrt{\frac{\pi}{s}}\, e^{-\frac{1}{4s}}$ **6**

(c) Obtain Fourier series for the function **8**

$$f(x) = \begin{cases} 1 + \frac{2x}{\pi} & -\pi \le x \le 0 \\ 1 - \frac{2x}{\pi} & 0 \le x \le \pi \end{cases}$$

Deduce that $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots\ldots\ldots\ldots$

3. (a) Find non singular matrices P and Q such that PAQ is in normal form. Also find their ranks. $A = \begin{bmatrix} 2 & 1 & 4 & 3 \\ 1 & 0 & 2 & 2 \\ 4 & 1 & 9 & 7 \end{bmatrix}$ 6

(b) Prove that cross-ratio remains invariant under bilinear transformation. 6

(c) Solve using Laplace Transforms $\frac{d^2y}{dt^2} + 9y = 18t$ given $y(0) = 0$ and $y\left(\frac{\pi}{2}\right) = 0$. 8

[TURN OVER

4. (a) Using convolution theorem, find $L^{-1}\left[\frac{1}{(s^2+4s+13)^2}\right]$. 6

(b) Find eigen values and eigen vectors of the matrix $\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$. 6

(c) Find Fourier series for f(x) in $(0, 2\pi)$ 8

$$f(x) = \begin{cases} x & 0 \le x \le \pi \\ 2\pi - x & \pi < x \le 2\pi \end{cases}$$

Hence deduce that $\frac{\pi^4}{96} = \frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \ldots\ldots\ldots\ldots$

5. (a) Verify Cayley-Hamilton theorem for the matrix A and hence find A^{-1}, where 6

$$A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$$

(b) Evaluate $\int_0^{1+i} z^2 dz$ along the parabola $x = y^2$. 6

(c) If $f(z) = u + iv$ in analytic and $u + v = \frac{2\sin(2x)}{(e^{2y} + e^{-2y}) - 2\cos(2x)}$. Find f(z). 8

6. (a) Find $L^{-1}\left[\mathrm{Tan}^{-1}\left(\frac{2}{s^2}\right)\right]$ 6

(b) Express the matrix $A = \begin{bmatrix} 1+3i & 2 & 3i \\ -2i & 0 & 1+2i \\ 4 & 2+5i & -i \end{bmatrix}$ as the sum of Hermitian and skew Hermitian matrix. 6

(c) Find the bilinear transformation under which 1, +i, –i from the z-plane is mapped onto 0, 1, ∞ of w-plane. Further show that under this transformation the unit circle in w-plane is mapped onto a straight line in the z-plane. Write the name of the this line. 8

7. (a) Find $L^{-1}\left[\frac{s^2+2s+3}{(s^2+2s+5)(s^2+2s+2)}\right]$ 6

(b) Find the orthogonal trajectory of the family of curves $e^x \cos y - xy = c$ 6

(c) Expand $f(x) = lx - x^2 \quad 0 < x < l$ in a half range sine series and hence deduce that 8

$$\frac{\pi^3}{3^2} = \frac{1}{1^3} - \frac{1}{3^3} + \frac{1}{5^3} - \frac{1}{7^3} + \ldots\ldots\ldots\ldots .$$
