Sem III / mech / 18-06-10 / Applied maths-III

Con. 3161-10. (REVISED COURSE) AN-2443

(3 Hours) [Total Marks : 100

1. (i) Using adjoint find the inverse of $A = \begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & 2 \end{bmatrix}$ 05

(ii) Determine whether the function $f(z) = \cosh z$ is analytic or not. If so, find the derivatives. 05

(iii) Prove: If X is an Eigen vector of A then X cannot correspond more than one Eigen value of A. 05

(iv) Evaluate: $\int_0^{2+i} \left(\bar{z}\right)^2 dz$ along, the line $x = 2y$ 05

2. (i) Find two non-singular matrices P & Q such that PAQ is in normal form for, $A = \begin{bmatrix} 2 & 1 & 4 \\ 3 & 2 & 2 \\ 7 & 4 & 10 \\ 1 & 0 & 6 \end{bmatrix}$ 06

(ii) If $f(z) = u + iv$ is an analytic function, prove that 06

$$\left\{\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right\}|f(z)|^2 = 4|f'(z)|^2$$

(iii) Find; 08

(a) $L\{J_0(t)\} \text{ Where } J_0(t) = \sum_0^{\infty} \frac{(-1)^r}{(r!)^2}\left(\frac{t}{2}\right)^{2r}$ (b) $L\left\{\sinh\frac{1}{2}t.\sin\frac{\sqrt{3}}{2}t\right\}$

3. (i) Find the eigen values and the eigen vectors of A^2 where 06

$$A = \begin{bmatrix} 3 & 1 & -1 \\ 2 & 2 & -1 \\ 2 & 2 & 0 \end{bmatrix}$$

(ii) $\int_C \frac{\cos \pi z}{z^2 - 1} dz$ where C is the circle $|z + 1 - i| = 2$ 06

(iii) Find; 08

(a) $L^{-1}\left\{\frac{5s^2 - 15s - 1}{(s+1)(s-2)^2}\right\}$ (b) $L^{-1}\left\{\frac{s e^{-4\pi s/5}}{s^2 + 4}\right\}$

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4. (i) Prove that every hermitian matrix A can be expressed as $B + iC$ where B is real symmetric and C is real skew-symmetric matrix 06

(ii) Use Cayley-Hamilton Thm. To find $2A^5 - 3A^4 + A^2 - 4I$ 06

Where $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

(iii) Find Laurent's expansion indicating region of convergence. 08

$$\frac{z}{(z-1)(z-2)} \text{ about } z = -2 \text{ and } z = 0$$

5. (i) Find the analytic function whose imaginary part is, 06

$$v = \log(x^2 + y^2) + x - 2y$$

(ii) Find the residues of $f(z) = z^3 e^{\frac{1}{z}}$ functions using Laurent's expansion. 06

(iii) Find the values of λ and μ such that the following equations , 08

$$2x + 3y + 5z = 9$$
$$7x + 3y - 2z = 8$$
$$2x + 3y + \lambda z = \mu$$

such that the above system (a) no solution (b) a unique solution (c) an infinite number of solutions.

6. (i) Find $L^{-1}\left\{\frac{s^2}{(s^2+a^2)(s^2+b^2)}\right\}$ using convolution Thm. 06

(ii) If (l_r, m_r, n_r) $r = 1,2,3$ are the direction cosines of three mutually perpendicular lines prove that the following matrix 06

$$\begin{bmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{bmatrix} \text{ is orthogonal.}$$

(iii) Evaluate: $\int_0^{2\pi} \frac{d\theta}{(2+\cos\theta)^2}$ 08

7. (i) Solve : $3\frac{dy}{dt} + 2y = e^{3t}$ where $y = 1$ at $t = 0$ 06

(ii) If $A = \begin{bmatrix} 3 & -2 \\ 4 & -3 \end{bmatrix}$ Find $3A^{57} + 2A^{18}$ 06

(iii) Find the bilinear transformation which maps the points 2, i, −2 onto the points 1, i, −1.and also find its fixed points. 08

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