

(3 Hours)

N.B. (1) Question No. 1 is compulsory.(2) Attempt any **four** out of remaining **six** questions.(3) **Figures** to the **right** indicate **full** marks.(4) Make **suitable** assumptions if **required** and justify the same.

1. (a) Find the orthogonal trajectory of the family of curves $x^3y - xy^3 = c$ 5
- (b) Find the Laplace transform of — 5
- (i) $\sin\sqrt{t}$ (ii) $\frac{\cos\sqrt{t}}{\sqrt{t}}$
- (c) Show that $[\text{diag}(\alpha \ \beta \ \gamma)]^{-1} = \text{diag}\left(\frac{1}{\alpha}, \frac{1}{\beta}, \frac{1}{\gamma}\right)$ if $\alpha \ \beta \ \gamma \neq 0$. 5
- (d) If $f(x) = x - x^2$ for $-1 < x < 1$ obtain Fourier series of $f(x)$ in $(-1, 1)$. 5
2. (a) Find the Fourier series for the function $f(x) = |x|$ in $(-\pi, \pi)$ and hence deduce 6
- that $\frac{\pi^2}{8} = 1 + \frac{1}{3^2} + \frac{1}{5^2} + \dots$
- (b) Show that $u = \cos x \cosh y$ is a harmonic function. Find its harmonic conjugate 7
- and corresponding analytic function.
- (c) Determine l, m, n and find A^{-1} if $A = \begin{bmatrix} 0 & 2m & n \\ / & m & -n \\ / & -m & n \end{bmatrix}$ is orthogonal. 7
3. (a) Show that $\int_C \log z dz = 2\pi i$ where C is the unit circle in the z plane. 6
- (b) Using Convolution Theorem, find the inverse Laplace transform of — 7
- $$\frac{s^2 + 2s + 3}{(s^2 + 2s + 2)(s^2 + 2s + 5)}$$
- (c) Show that the set of function $\cos nx$ is orthogonal function in $(0, 2\pi)$. 7
4. (a) Find the Fourier series for the function $f(x) = x$ in $(-\pi, \pi)$ and hence deduce 6
- that $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots = \frac{\pi}{4}$
- (b) Find an analytic function whose real part is $\frac{1}{2} \log(x^2 + y^2)$. Also find its imaginary 7
- part using Milne Thompson's method.
- (c) Using partial fraction method find the inverse Laplace transform of $\frac{s+2}{s^2(s+3)}$. 7

[TURN OVER

5. (a) If $f(x) = c_1 \phi_1(x) + c_2 \phi_2(x) + c_3 \phi_3(x)$ where c_1, c_2, c_3 are constant and ϕ_1, ϕ_2 and ϕ_3 6

are orthonormal sets of function on (a, b) . Then show that $\int_a^b [f(x)]^2 dx = c_1^2 + c_2^2 + c_3^2$

- (b) Find the adjoint of the coefficient matrix of the following system and determine its inverse matrix. Hence solve the system :— 8

$$x + y + z = \alpha$$

$$x + 2y + 3z = \beta$$

$$2x + 3y + 8z = \gamma$$

- (c) Expand a function $\cos z$ in a Taylor's series about $z = \frac{\pi}{4}$. 6

6. (a) If $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & -1 \\ 3 & 1 & 1 \end{bmatrix}$, find two matrices P and Q such that PAQ is in normal form. 6

- (b) Evaluate the following using Cauchy Residue Theorem — 7

(i) $\int \frac{(z+4)^2}{z^4 + 5z^3 + 6z^2} dz$ where C is $|z|=1$

(ii) $\int \operatorname{cosec} z dz$ where C is $|z|=1$

- (c) Prove that necessary and sufficient condition for a square matrix A to possess the inverse is that $|A| \neq 0$. 7

7. (a) Solve : $2x - y + 3z = 0$ 6

$$3x + 2y + z = 0$$

$$x - 4y + 5z = 0$$

- (b) Find the Laplace transform of — 8

(i) $e^{4t} \sin^{3t}$ (ii) \sin^{4t} (iii) $(t \sinh_2 t)^2$

- (c) Solve the equation by Laplace transform method :— 6

$$\frac{d^2x}{dt^2} + 4x = 0$$

given $x(0) = 1, x'(0) = -2$