Sem III / Inst. / 18-06-10 / Engg. maths-III

Con. 3037–10. **AN–2462**

(3 Hours) **[Total Marks : 100**

N.B. : (1) Question No. **1** is **compulsory**.
(2) Attempt any **four** questions from remaining **six** questions.
(3) **Figures** to **right** indicate **full** marks.

1. (a) Using Laplace transform, find the solution of the system – **5**

$$\frac{dx}{dt} + y = 1, \quad x + \frac{dy}{dt} = 0$$

Satisfying $x(0) = 1$ and $x(t) \to 0$ as $t \to \infty$.

(b) Expand f(t) in a Fourier cosine series and draw the corresponding periodic extension **5**

of f(t), where $f(t) = \begin{cases} 0 & \text{for } 0 < t < \pi/2 \\ 1 & \text{for } \pi/2 < t < \pi \end{cases}$

(c) Examine the effect of inversion $w = \frac{1}{2}$ on the line $\mathrm{Re}(z) = a$ (a nonzero constant). **5**

(d) Use convolution theorem to find **5**

$$L^{-1}\left[\frac{1}{(s-s)\left[(s-3)^2+8\right]}\right]$$

2. (a) Discuss the transformation $w = \sin h\,(z)$. **5**

(b) Find the Fourier expansion of **7**

$f(t) = t - t^2$ in $0 < t < 1$

(c) Prove that **8**

$$L\left[t^{-1/2}\cos\left(a t^{1/2}\right); p\right] = \left(\frac{\pi}{p}\right)^{1/2} e^{-a^2/4p}$$

and deduce that

(i) $L\left[t^{-1}\sin\left(a t^{1/2}\right); p\right] = \mathrm{Erf}\left[\frac{1}{2} a p^{-1/2}\right]$

(ii) $L\left[\sin\left(a b^{1/2}\right); p\right] = \frac{1}{2} a \left(\frac{\pi}{p^3}\right)^{1/2} e^{-a^2/4p}$

3. (a) Find the Laurent series of $\frac{1}{z(z-1)^2}$ about **8**

(i) $0 < |z| < 1$ (ii) $|z| > 1$ (iii) $0 < |z-1| < 1$ (iv) $|z-1| > |$

(b) Expand $f(x) = \begin{cases} 0 & \text{if } -L \le x \le 0 \\ L & \text{if } 0 < x \le L \end{cases}$ **8**

in Fourier series and choose x appropriately to sum the series

$$\sum_{1}^{\infty} \frac{(-1)^{n+1}}{2n-1}$$

4. (a) Calculate the residue at the singular point of 8

$$f(z) = \frac{\sin 3z - 3\sin z}{(\sin z - z)\sin z}$$

(b) Evaluate $\displaystyle\int_{c:|z|=2} \frac{dz}{z(z-1)^2(z+3)}$ 6

(c) Show that $\displaystyle\int_0^{\infty} \frac{\cos\lambda x}{\lambda^2+1}\, d\lambda = \frac{\pi}{2} e^{x},\ x \geq 0$ by 6

the definition of Fourier cosine integral.

5. (a) Evaluate $\displaystyle\int_{c:|z|=1} \frac{\mathrm{Re}(z)\,dz}{z-\alpha},\ 0<|\alpha|<1$ 6

(b) Evaluate $\displaystyle\int_{r} f(z)\,dz,\ f(z) = z^n$ where $r = e^{it}$, 6

$0 \leq t \leq 2\pi$, and n is any integer

(c) Find Laplace inverse for the following function 8

(i) $\dfrac{2s^2 - s}{\left(s^2+4\right)^2}$ (ii) $\dfrac{s+b}{s^2 + 4s + 12}$.

6. (a) Find the complex form of Fourier series of $f(x) = e^{ax}$ $(-\pi < x < \pi)$ in the form 6

$$e^{ax} = \frac{\sinh a\pi}{\pi} \sum_{n=-\infty}^{\infty} (-1)^n \frac{a+in}{a^2+n^2} e^{inx}$$

(b) Use the Heariside formula to find 8

$$L^{-1}\left[\frac{3s}{\left(s^2-2s+5\right)(s+1)(s-5)}\right]$$

(c) Expand $f(z) = \dfrac{1}{1+z}$ about $-2i$ 6

7. (a) Define orthogonal and orthonormal set of functions. Show that $\{\cos(nx)\}$ 6
n = 1, 2, ... is orthogonal over $[-\pi, \pi]$.

(b) Evaluate (i) $\displaystyle\int_0^{2\pi} \frac{d\theta}{13 + 5\cos\theta}$ 8

(ii) $\displaystyle\int_{-\infty}^{\infty} \frac{dx}{\left(x^2+1\right)\left(x^2+4\right)^2}$

(c) Find the bilinear transformation which maps the points $z = 1, -1, \infty$ onto the points 6