sem III / 18-06-10 / E/H / Applied maths - III

Con. 3867-10. (REVISED COURSE) AN–2548

(3 Hours) [Total Marks : 100

N.B. (i) Question **No.1** is compulsory
(ii) Answer any **four** out of the remaining six questions
(iii) Figures to the right indicate full marks

1. (a) Find the Laplace transform of $\int_0^t u\cos^2 u\,du$. [5]

(b) Show that every skew Hermitian matrix can be expressed as $P+iQ$ where P is a real skew symmetric matrix and Q is a real symmetric matrix. [5]

(c) Find the Z transforms of (i) $\delta(n-k)$ where $\delta(k)=\begin{cases}1, & k=0\\ 0 & \text{otherwise}\end{cases}$ and

(ii) $U(k)=\begin{cases}1, & k\geq 0\\ 0 & \text{otherwise}\end{cases}$. [5]

(d) Obtain the Fourier series of

$$f(x)=\begin{cases}0, & -\pi\leq x\leq 0\\ x^2, & 0\leq x\leq \pi\end{cases}$$

where $f(x)$ is periodic with period 2π. [5]

2. (a) Find non-singular matrices P and Q such that the normal form of :–

$$A=\begin{bmatrix}1 & 3 & 6 & -1\\ 1 & 4 & 5 & 1\\ 1 & 5 & 4 & 3\end{bmatrix}$$ is PAQ. What is the rank of A? [7]

(b) Obtain the inverse Laplace transforms of the following:

(i) $\dfrac{3s+7}{s^2-2s-3}$ (ii) $\dfrac{s+2}{(s+3)(s+1)^3}$ [7]

(c) Find the Fourier integral representation of the function:

$$f(x)=\begin{cases}0, & x<0\\ \dfrac{1}{2}, & x=0\\ e^{-x}, & x>0.\end{cases}$$ [6]

3. (a) Find $\int_0^\infty \cos(tx^2)\,dx$ and hence evaluate $\int_0^\infty \cos x^2\,dx$ [7]

(b) Find the inverse Z transform of (i) $(z-5)^{-3}$ when $|z|>5$

(ii) $\dfrac{z^2}{(z-1)(z-\frac{1}{2})}$, for $1/2<|z|<1$. [7]

(c) Define an orthogonal matrix. Is $A=\begin{bmatrix} \cos a\cos b & \sin b & \cos b\sin a \\ -\sin b\cos a & \cos b & -\sin b\sin a \\ -\sin a & \cos a & 0 \end{bmatrix}$ orthogonal? [6]

4. (a) (i) Solve by the Gauss-Jordan method:

$$3x+2y-2z = 11$$
$$x-2y+3z = -9$$
$$2x+3y+4z = 0$$

(ii) Solve by the Gauss-Seidel method: (Go up to 3 iterations):

$$10x+y+z = 10$$
$$2x+10y+z = 11$$
$$2x+2y+10z = -6.$$

[3+4]

(b) Obtain the Fourier series of $f(x)=\sqrt{1-\cos x}$ in the interval $(0,2\pi)$.

Deduce that $\frac{1}{2}=\sum_{n=1}^{\infty}\frac{1}{4n^2-1}$ [7]

(c) Find the Fourier transform of: $f(x)=e^{-|x|}$ [6]

5. (a) Find the half-range sine series for $f(x)=\frac{\pi}{4}$ in $(0,\pi)$.

Deduce that

(i) $\frac{\pi}{4}(\frac{\pi}{2}-x)=\frac{1}{1^2}\cos x+\frac{1}{3^2}\cos 3x+\frac{1}{5^2}\cos 5x+\ldots$

(ii) $\frac{\pi}{8}x(\pi-x)=\frac{1}{1^3}\sin x+\frac{1}{3^3}\sin 3x+\frac{1}{5^3}\sin 5x+\ldots$ [7]

(b) Solve using Laplace transforms:

$$\frac{d^2y}{dt^2}-\frac{dy}{dt}-2y=20\sin t,\ y(0)=1 \ \text{and}\ y'(0)=2.$$ [7]

(c) Discuss for what values of λ and μ, the following system of equations :

$$x+y+z = 6$$
$$x+2y+3z = 10$$
$$x+2y+\lambda z = \mu$$

has (i) no solution (ii) a unique solution and (iii) infinite number of solutions. [6]

6. (a) State the convolution theorem for Z-transforms. Use the theorem to find $Z(h(k))$ where $h(k)$ is the convolution of $f_1(k)=\frac{1}{2^k}$, $k\geq 0$ and

$f_2(k) = \cos k\pi$, $k\geq 0$ [7]

(b) (i) Express $f(t)=\begin{cases}2t, & 0<t<1\\ 3t^2 & t>1\end{cases}$ in terms of Heaviside's unit step function and find its Laplace Transform.

(ii) Evaluate using Laplace Transform: $\int_0^{\infty}e^{-t}(1+2t-t^2+t^3)H(t-1)\,dt$ where $H(t)$ is the Heaviside's unit step function. [7]

(c) Find the Fourier series of $f(x)=x|x|$ in (-1,1). [6]

7. (a) Find the Fourier sine transform of $f(x)=\frac{e^{-x}}{x}$. Hence evaluate :–

$$\int_0^\infty \tan^{-1}\left(\frac{x}{a}\right)\sin x\,dx.$$ [7]

(b) Obtain the Laplace transform of (i) $t^2e^t\sin 4t$ (ii) $\frac{\cosh 2t \sin 2t}{t}$ [7]

(c) When do you say that vectors $X_1, X_2, \ldots, X_n$ are linearly dependent? Are the vectors $X_1=[1\ 3\ 4\ 2]$, $X_2=[2\ -1\ 3\ 2]$, $X_3=[3\ -5\ 2\ 2]$ linearly dependent? [6]