

N.B. (1) Question No 1 is compulsory

(2) Attempt any four questions from the remaining six questions.

(3) Figures to the right indicate full marks.

1.

a) Change the order of the integration of $\int_0^{2a} \int_{\sqrt{2ax-x^2}}^{\sqrt{2ax}} f(x, y) dx dy$ 5

b) Solve $\left[\frac{\log(\log y)}{x} + \frac{2}{3} xy^3 \right] dx + \left[\frac{\log x}{y \log y} + x^2 y^2 \right] dy = 0$ 5

c) Find the area of the hypocycloid $x = a \cos^3 \theta, y = b \sin^3 \theta$ 5

d) Evaluate $\int_0^{\infty} x^n e^{-\sqrt{ax}} dx$ 5

2.

a) Solve $[3x^2 y^4 + 2xy] dx + [2x^3 y^3 - x^2] dy = 0$ 6

b) Evaluate $\iint_R xy dx dy$ over the region R given by $x^2 + y^2 - 2x = 0, y^2 = 2x, y = x$. 7

c) Evaluate $\int_0^{\frac{\pi}{2}} \frac{dx}{a^2 \sin^2 x + b^2 \cos^2 x}$ and show that

$$\int_0^{\frac{\pi}{2}} \frac{dx}{(a^2 \sin^2 x + b^2 \cos^2 x)^2} = \frac{\pi}{4ab} \left(\frac{1}{a^2} + \frac{1}{b^2} \right)$$
 7

3.

a) Solve $(x^2 - 1) \sin x \frac{dy}{dx} + [2x \sin x + (x^2 - 1) \cos x] y = (x^2 - 1) \cos x$ 6

b) By changing into polar coordinates evaluate $\iint \frac{4xy}{x^2 + y^2} e^{-x^2 - y^2} dx dy$ over the region bounded by the circle $x^2 + y^2 - x = 0$ in the first quadrant. 7

c) Prove that $\int_0^{\frac{\pi}{2}} \frac{d\theta}{\sqrt{1 - \frac{1}{2} \sin^2 \theta}} = \frac{\left(\Gamma \frac{1}{4} \right)^2}{4\sqrt{\pi}}$ 7

[TURN OVER]

4.

- a) Evaluate $\int_1^e \int_1^y \int_1^{e^x} \log z \, dx dy dz$ 6
- b) Solve $(D-1)^2(D^2+1)y = e^x + \sin^2 \frac{x}{2}$ 7
- c) Solve the differential equation $\frac{dy}{dx} = \frac{1}{x+y}$, $x_0=0, y_0=1$ for the interval $(0,1)$ choosing $h=0.5$ by Runga Kutta Method of Fourth Order. 7

5.

- a) Solve $\frac{dy}{dx} = 1 + y^2$ with initial conditions $x_0=0, y_0=0$ by Taylor's method. Obtain y as a series in powers of x . Hence find the approximate values of y for $x=0.2, 0.4$. 6
- b) By using method of variation of parameters solve $(D^2 - 2D + 4)y = e^{2x} \sec^2 x$ 7
- c) Evaluate $\iiint dx dy dz$ over the volume of the tetrahedron bounded by $x=0, y=0, z=0$ and $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ 7

6.

- a) Find the length of arc of $r=a(1-\cos\theta)$ lying outside the circle $r=a\cos\theta$. 7
- b) Solve $(D^2 - D - 2)y = 2 \log x + \frac{1}{x} + \frac{1}{x^2}$ 7
- c) Evaluate $\iint r^2 \sin \theta dr d\theta$ over the cardioid $r = a(1 + \cos \theta)$ above the initial line. 7

7.

- a) Solve $x^2 \frac{d^2 y}{dx^2} - x \frac{dy}{dx} + 4y = \cos \log x + x \sin \log x$ 6
- b) Find the mass of the lamina over the area bounded by the curves $16y^2 = x^3$ and the line $2y=x$, if density at any point varies as the distance of the point from x axis. 7
- c) Solve $\frac{di}{dt} + \frac{R}{L} i = \frac{E}{L}$ for the case in which the circuit has initial current i_0 at time $t=0$ and the emf impressed is given by $E = E_0 e^{-kt}$ 7