Sem I / Rev / App Maths-I / May 10

1st-half-NK-10BB 57

Con. 2648-10. **(REVISED COURSE)** **AN-9752**

(3 Hours) **[Total Marks : 100**

N.B. (1) Question No. **1** is **compulsory**.
(2) Attempt any **four** questions from remaining **six** questions.
(3) **Figures** to the **right** indicate the **full** marks.
(4) Assume the **suitable** data if **needed** with justification.

1. (a) If $|z-1| < |z+1|$ prove that $\text{Re } z > 0$ 5
   (b) If $u = \log(\tan x + \tan y + \tan z)$. Prove that— 5

$$\sin 2x \frac{\partial u}{\partial x} + \sin 2y \frac{\partial u}{\partial y} + \sin 2z \frac{\partial u}{\partial z} = 2.$$

   (c) Find $\phi(r)$ s.t. $\nabla\phi = \frac{\bar{r}}{r^5}$ and $\phi(1) = 0$. 5

   (d) Prove that : $e^{x\cos x} = 1 + x + \frac{x^2}{2} + \ldots.$ 5

2. (a) If $x + \frac{1}{x} = 2\cos\theta$, $y + \frac{1}{y} = 2\cos\phi$ then show that— 6

$$x^2y^2 + \frac{1}{x^2y^2} = 2\cos(2\theta + 2\phi)$$

   (b) Prove : $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right) = \frac{1}{2}\left(x - \frac{x^3}{3} + \frac{x^5}{5} - \ldots\ldots\right)$ 6

   (c) Find all the stationary points of 8
$$f(x, y) = x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$$
   Determine which are maximum and minimum.

3. (a) If $a, a^2, a^3, \ldots\ldots, a^6$ are the roots of $x^7 - 1 = 0$ prove that 6
   $(1-a)(1-a^2)\ldots\ldots(1-a^6) = 7$
   (b) Test the convergence of — 6

$$\frac{3+4}{4+5}, \frac{3^2+4^2}{4^2+5^2}, \frac{3^3+4^3}{4^3+5^3}, \ldots\ldots$$

   (c) Verify : $(\bar{a}\times\bar{b})\times\bar{c} = (\bar{a}\cdot\bar{c})\bar{b} - (\bar{b}\cdot\bar{c})\bar{a}$ and 8

$$\bar{a}\times(\bar{b}\times\bar{c}) = (\bar{a}\cdot\bar{c})\bar{b} - (\bar{a}\cdot\bar{b})\bar{c}$$

   For $\bar{a} = 3i - 2j + 2k$; $\bar{b} = 6i + 4j + 2k$; $\bar{c} = 3i + 2j + 4k$

[TURN OVER

4. (a) If $x = e^u \tan v$, $y = e^u \sec v$ find, $\left(x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}\right) \cdot \left(x\frac{\partial v}{\partial x} + y\frac{\partial v}{\partial y}\right)$ 6

(b) Prove that the equation $2x^3 - 3x^2 - x + 1 = 0$ has one root between 1 and 2. 6

(c) Show that $\log\left(e^{i\alpha} + e^{i\beta}\right) = \log 2\cos\left(\frac{\alpha - \beta}{2}\right) + i\left(\frac{\alpha + \beta}{2}\right)$. 8

5. (a) State and prove Euler's Thm. For function of two variables. 6

(b) If $\sin(\theta + i\phi) = \tan\alpha + i\sec\alpha$ then show that— 6
$\cos 2\theta \cosh 2\phi = 3$.

(c) If $y = \cos^{-1} x$ prove that, 8
$$(1 - x^2)\, y_{n+2} - (2n + 1)\, x\, y_{n+1} - n^2 y_n = 0$$

6. (a) If $y = \frac{\log x}{x}$ prove that 6
$$y_5 = \frac{5!}{x^6}\left[1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} - \log x\right]$$

(b) Find the product of all values of $\left(\frac{1}{2} - \frac{i\sqrt{3}}{2}\right)^{\frac{3}{4}}$ 6

(c) If $u = x^3 \sin^{-1}\frac{y}{x} + x^4 \tan^{-1}\frac{y}{x}$ find the value of, 8
$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} + x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} \text{ at } x = 1 \text{ and } y = 1$$

7. (a) Find 'a', 'b', 'c' if $\lim_{x \to 0} \frac{x(a + b\cos x) - c\sin x}{x^5} = 1$ 6

(b) State the Lagrange's Thm and give its geometrical interpretation. 6

(c) If $u = x^y$ show that $\frac{\partial^3 u}{\partial x^2 \partial y} = \frac{\partial^3 u}{\partial x \partial y \partial x}$. 8
